දකුණු පළාත් අධ්‍යාපන දෙපාර්තමේන්තුව

උසස් පෙළ 2026

සංයුක්ත ගණිතය I

(අවසාන වාර පරීක්ෂණය) පිළිතුරු පත්‍රය

A කොටස.

(01) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} > 1$

$n=2$ විට ව.පැ $= 1 + \frac{1}{2} = \frac{3}{2} > 1$ $\therefore$ $n=2$ විට ප්‍රතිඵලය සත්‍ය වේ. (5)

$n=p$ විට $p \in \mathbb{Z}^+ - \{1\}$ ප්‍රතිඵලය සත්‍ය යැයි ගනිමු.

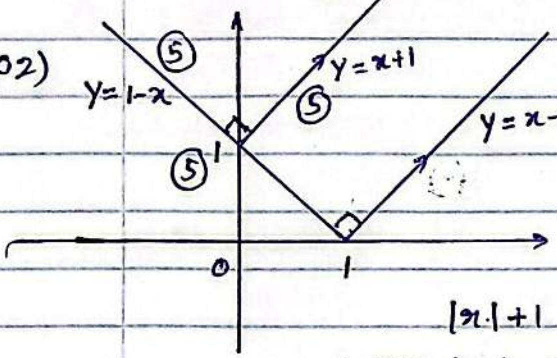
$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{p} > 1$ (5)

$n=p+1$ විට

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{p} + \frac{1}{p+1} > 1 + \frac{1}{p+1} > 1$, $\because p+1 > 2$, $0 < \frac{1}{p+1} < \frac{1}{2}$ (5) (5)

$\therefore$ $n=p+1$ වද ප්‍රතිඵලය සත්‍ය වේ. එනම් වාරම, $n \in \mathbb{Z}^+ - \{1\}$ සඳහා ගණිත අභ්‍යුහන මූලධර්මයට අනුව ප්‍රතිඵලය සත්‍ය වේ. (5) $\boxed{25}$

(02)



$|x|+1 \le |1-x|$ වන x හි අගයන් යනු $y=|x|+1$ ප්‍රස්තාරය $y=|1-x|$ ප්‍රස්තාරයට පහළින් හෝ මත පිහිටන x අගයන් වේ.

$\therefore \{x \mid x \in \mathbb{R}, (-\infty, 0]\}$ (5)

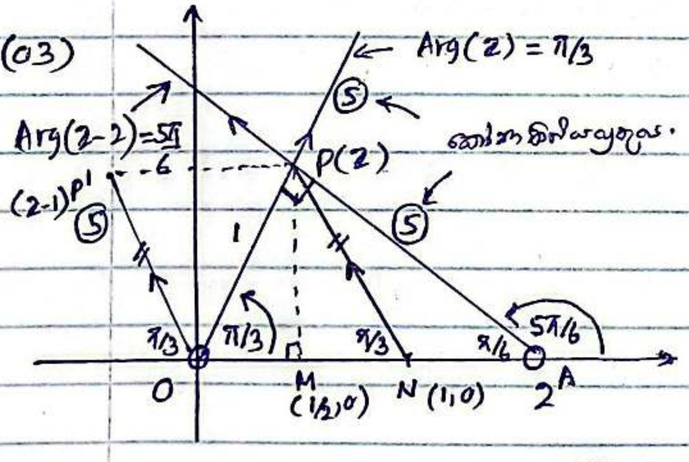
$|x|+1 \le |1-x|$ හි විසඳුම $-\infty < x \le 0$

$x = x+1 \Rightarrow |x+1|+1 \le |1-(x+1)|$ හි විසඳුම $-\infty < x+1 \le 0$

$|x+1|+1 \le |x|$ හි විසඳුම $-\infty < x \le -1$ වේ. (5)

$\boxed{25}$

(03)



ජ්‍යාමිතියෙන්. $OP = OA \cos \pi/3 = 2 \cdot \frac{1}{2} = 1$

$\left.\begin{aligned} OM &= 1 \cos \pi/3 = \tfrac{1}{2} \\ MP &= 1 \sin \pi/3 = \sqrt{3}/2 \end{aligned}\right\}$ (5)

$\therefore P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $\therefore z = \frac{1}{2} + \frac{\sqrt{3}}{2} i$ (5)

$|z-1| = PN$

$NP =$ හා OP' වන පරිදි P' ලක්ෂ්‍යය මගින් $(z-1)$ සංකීර්ණ සංඛ්‍යාව නිරූපණය වේ.

$\boxed{25}$

(04) $\displaystyle \lim_{x\to\pi} \frac{[\sin 4x - \sin 2x](\sqrt{x}-\sqrt{\pi})}{(\pi-x)^2} = \lim_{x\to\pi} \frac{2\cos 3x \sin x\,(\sqrt{x}-\sqrt{\pi})}{(\pi-x)\;(\pi-x)}$ (5)

$\displaystyle = -2\lim_{x\to\pi}\cos 3x \left(\lim_{\pi-x\to 0}\frac{\sin(\pi-x)}{(\pi-x)}\right)^{\to 1} \lim_{x\to\pi}\frac{x^{1/2}-\pi^{1/2}}{x-\pi}$ (5) (5) (5)

$\displaystyle = -2\underbrace{\cos 3\pi}_{-1}\cdot 1\cdot \frac{1}{2}\pi^{-1/2} = \frac{1}{\sqrt{\pi}}$ (5) $\boxed{25}$

(05) $x^2 - y^2 = 1$

$2x - 2y\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{x}{y} \Rightarrow \left.\dfrac{dy}{dx}\right|_A = \dfrac{\sec\theta}{\tan\theta}$

A හිදී ස්පර්ශකයේ සමීකරණය $\dfrac{\sec\theta}{\tan\theta} = \dfrac{y-\tan\theta}{x-\sec\theta}$ (5)

$x\sec\theta - \sec^2\theta = y\tan\theta - \tan^2\theta \Rightarrow x\sec\theta - y\tan\theta = \sec^2\theta - \tan^2\theta = 1$ — ①

$(\sqrt{2},1)$ ලක්ෂ්‍යය සඳහා $\theta = \frac{\pi}{4}$, ① $\Rightarrow$ $x\sec\frac{\pi}{4} - y\tan\frac{\pi}{4} = 1 \Rightarrow \sqrt{2}x - y = 1$ (5)

$(\sqrt{2},1)$ ලක්ෂ්‍යයේදී ස්පර්ශකයේ සමීකරණය

මෙම ස්පර්ශකය මත $\left(\dfrac{1+t^2}{1-t^2}, \dfrac{2t}{1-t^2}\right)$ පවතින නිසා,

$\sqrt{2}\left(\dfrac{1+t^2}{1-t^2}\right) - \dfrac{2t}{1-t^2} = 1$ (5) $\Rightarrow \sqrt{2} + \sqrt{2}t^2 - 2t = 1 - t^2$

$(\sqrt{2}+1)t^2 - 2t + \sqrt{2} - 1 = 0$ (5)

$t^2 - \dfrac{2(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}t + \dfrac{(\sqrt{2}-1)^2}{(\sqrt{2}+1)(\sqrt{2}-1)} = 0$

$t^2 - 2(\sqrt{2}-1)t + (\sqrt{2}-1)^2 = 0 \Rightarrow \{t - (\sqrt{2}-1)\}^2 = 0$

$\therefore\ t = \sqrt{2}-1$ (5) ඒකීය ලෙස ලැබේ

$\boxed{25}$

(06) $\displaystyle (\sqrt{x}+\sqrt{2})^8 = \sum_{r=0}^{8} {}^8C_r\, x^{\frac{8-r}{2}} \sqrt{2}^{\,r}$ (5), $\displaystyle {}^8C_r = \frac{\lfloor 8}{\lfloor r \lfloor 8-r}$

x^m පද සඳහා $\dfrac{8-r}{2} = m$ (5), $m = 0,1,2,3,4$ එවිට $r = 8,6,4,2,0$ වේ (5)

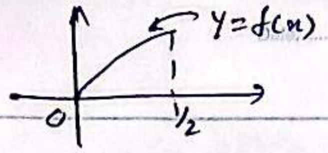
x^m සංගුණකවල එකතුව $= {}^8C_8\sqrt{2}^8 + {}^8C_6\sqrt{2}^6 + {}^8C_4\sqrt{2}^4 + {}^8C_2\sqrt{2}^2 + {}^8C_0\sqrt{2}^0$ (5)

$= 16 + 28(8) + 70(4) + 28(2) + 1$

$= 16 + 224 + 280 + 56 + 1$ (5)

x^m සංගුණකවල එකතුව $= 577$ $\boxed{25}$

(07) $0 < x < 1$ සඳහා $y = \sqrt{\dfrac{x}{1-x}} > 0$



පරිමාව $= \int_0^{1/2} \pi\left(\dfrac{x}{1-x}\right) dx$ (5)

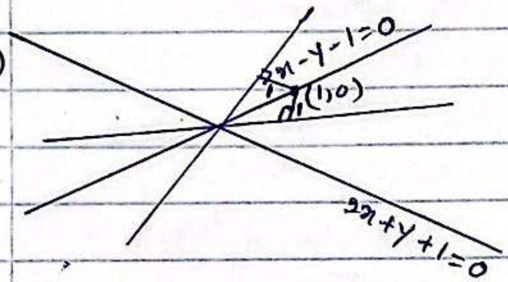
$= -\pi \int_0^{1/2} \dfrac{x}{x-1}\, dx = -\pi \int_0^{1/2} \dfrac{(x-1)+1}{x-1}\, dx$ (5)

$= -\pi \int_0^{1/2} \left[1 + \dfrac{1}{x-1}\right] dx$ (5) $= -\pi \left[x + \ln|x-1|\right]_0^{1/2}$ (5)

$= -\pi\left[\dfrac{1}{2} + \ln\dfrac{1}{2} - (0 + \underbrace{\ln 1}_{0})\right] = -\pi\left[\dfrac{1}{2} - \ln 2\right]$

පරිමාව $= \dfrac{\pi}{2}(2\ln 2 - 1)$ (5) $\boxed{25}$

(08.)



අවශ්‍ය රේඛාවක් $2x+y+1+\lambda(x-y-1)=0$ ලෙස ගනිමු. (5)

$(2+\lambda)x+(1-\lambda)y+1-\lambda=0$ —(A)

මෙම රේඛාවට $(1,0)$ සිට දුර $= 1$

$\dfrac{|2+\lambda+1-\lambda|}{\sqrt{(2+\lambda)^2+(1-\lambda)^2}} = 1$ (5)

$3^2 = (2+\lambda)^2+(1-\lambda)^2$

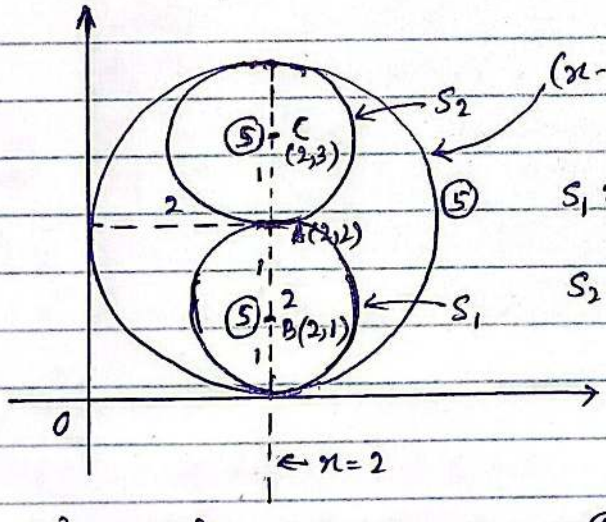
$2\lambda^2+2\lambda-4=0 \Rightarrow \lambda^2+\lambda-2=0$ (5)

$(\lambda-1)(\lambda+2)=0 \Rightarrow \lambda=1$ හෝ $\lambda=-2$ (5)

$\lambda=1$, (A) $\Rightarrow$ අවශ්‍ය රේඛාවක්, $3x=0 \Rightarrow x=0$ (5)

$\lambda=-2$, (A) $\Rightarrow$ අනෙක් රේඛාව, $3y+3=0 \Rightarrow y=-1$ $\boxed{25}$

(09.)



අවශ්‍ය වෘත්තවල සමීකරණ

$S_1: (x-2)^2+(y-1)^2=1$ (5)

$S_2: (x-2)^2+(y-3)^2=1$ (5) $\boxed{25}$

(10.) $\cos^2 x - \sin^2 x + 2\sin x\cos x = 0$ —(A)

$\cos 2x + \sin 2x = 0 \Rightarrow \tan 2x = -1 = \tan(-\pi/4)$ (5)

$2x = n\pi - \dfrac{\pi}{4},\ n\in\mathbb{Z}$ —(B) (5)

$(0,\pi)$ තුළ විසඳුම්, $\dfrac{3\pi}{8}, \dfrac{7\pi}{8}$ (5)

$x = 2x$ ආදේශ (A) $\Rightarrow$ (5)

$\cos^2 2x - \sin^2 2x + 2\sin 2x\cos 2x = 0$

එවිට විසඳුම් (B) අනුව $4x = n\pi - \dfrac{\pi}{4}$ වේ.

$(0,\pi)$ තුළ විසඳුම්,

$x = \dfrac{3\pi}{16}, \dfrac{7\pi}{16}, \dfrac{11\pi}{16}$ වේ. (5)

$\boxed{25}$

B කොටස.

(11) (a) $ax^2+bx+c=0 \ , \ a\neq 0$ — ①

මෙහි මූල α' හා β' නිසා ① හා සමීකරණය $a(x-\alpha')(x-\beta')=0$ ලෙස ලියමු. ⑤

$$a\left(x^2-(\alpha'+\beta')x+\alpha'\beta'\right)=0 \quad \text{—②} \quad ⑤$$

① ≡ ② බැවින් සංගුණක සමාන කිරීමෙන්,

$$x: \ b=-a(\alpha'+\beta') \ ⑤ \Rightarrow \alpha'+\beta'=-\frac{b}{a}$$

$$x^0: \ c=a\alpha'\beta' \ ⑤ \Rightarrow \alpha'\beta'=\frac{c}{a}$$

$\boxed{25}$

$$\sqrt{a}\,x^2-\sqrt{b}\,x+\sqrt{c}=0 \ , \quad a,b,c>0$$

මූල තාත්වික වීමට $\Delta \geq 0$ විය යුතුය. ⑤

$$b-4\sqrt{a}\sqrt{c}\geq 0 \ ⑤ \Rightarrow b\geq 4\sqrt{ac}$$

$\boxed{10}$

$$\alpha+\beta=\frac{\sqrt{b}}{\sqrt{a}} \ , \quad \alpha\beta=\frac{\sqrt{c}}{\sqrt{a}}$$

$$(\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\beta \ ⑤ =(\alpha+\beta)^2-4\alpha\beta \ ⑤ =\frac{b}{a}-\frac{4\sqrt{c}}{\sqrt{a}} \ ⑤$$

$$(\alpha-\beta)^2=\frac{b-4\sqrt{ac}}{a} \Rightarrow |\alpha-\beta|=\sqrt{\frac{b-4\sqrt{ac}}{a}} \ ⑤$$

$\boxed{20}$

$$|\alpha^2-\beta^2|=|\alpha-\beta||\alpha+\beta|=\sqrt{\frac{b-4\sqrt{ac}}{a}}\cdot\frac{\sqrt{b}}{\sqrt{a}}=\frac{\sqrt{b^2-4b\sqrt{ac}}}{a} \ ⑤$$

මූල $|\alpha-\beta|$ හා $|\alpha^2-\beta^2|$ වන වර්ගජ සමීකරණය

$$\left(x-|\alpha-\beta|\right)\left(x-|\alpha^2-\beta^2|\right)=0 \ ⑤$$

$$x^2-\left(|\alpha-\beta|+|\alpha^2-\beta^2|\right)x+|\alpha-\beta||\alpha^2-\beta^2|=0 \ ⑤$$

$$⑤ \ x^2-\left(\frac{1}{\sqrt{a}}\sqrt{b-4\sqrt{ac}}+\frac{1}{a}\sqrt{b^2-4b\sqrt{ac}}\right)x+\sqrt{\frac{b-4\sqrt{ac}}{a}}\cdot\frac{\sqrt{b^2-4b\sqrt{ac}}}{a}=0$$

$$⑤ \ a^{3/2}x^2-\sqrt{a}\left(\sqrt{a}\sqrt{b-4\sqrt{ac}}+\sqrt{b^2-4b\sqrt{ac}}\right)x+\sqrt{b}\left(b-4\sqrt{ac}\right)=0$$

$$a^{3/2}x^2-\sqrt{a}\left(\sqrt{a}\sqrt{b-4\sqrt{ac}}+\sqrt{b}\sqrt{b-4\sqrt{ac}}\right)x+\sqrt{b}\left(b-4\sqrt{ac}\right)=0$$

$$⑤ \ a^{3/2}x^2-\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b-4\sqrt{ac}}\,x+\sqrt{b}\left(b-4\sqrt{ac}\right)=0 \quad \text{—Ⓐ}$$

$\boxed{30}$

Ⓐ හි මූල γ හා δ නම්, $x=\gamma$, හා $x=\delta$.

මූල $\frac{1}{\gamma}$ හා $\frac{1}{\delta}$ වන වර්ගජ සමීකරණයේ විචල්‍යය y නම් $y=\frac{1}{\gamma}$ හා $y=\frac{1}{\delta}$ ⑤

$y=\frac{1}{x}$ හා $y=\frac{1}{x}$ ⑤ $\therefore x=\frac{1}{y}$ හි $x=\gamma$ හා $x=\delta$ ⑤ තෘප්ත කරයි.

$$\therefore x=\frac{1}{y} \text{ ආදේශයෙන් } ⑤ \ Ⓐ \Rightarrow a^{3/2}\cdot\frac{1}{y^2}-\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)\left(b-4\sqrt{ac}\right)\frac{1}{y}+\sqrt{b}\left(b-4\sqrt{ac}\right)=0$$

$$\sqrt{b}\left(b-4\sqrt{ac}\right)y^2-\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)\left(b-4\sqrt{ac}\right)y+a^{3/2}=0 \ ⑤$$

$\boxed{25}$

(b) $f(x) = ax^4 + bx^3 + cx^2 + 1$

$f(1) = a+b+c+1 = -1$ (5)

$a+b+c = -2$ — (1)

$f(x)$, $(x-1)^3$ න් බෙදූ විට ලැබෙන ලබ්ධිය $\phi(x)$ ද ශේෂය $x-2$ ද නිසා

$f(x) \equiv (x-1)^3\phi(x) + x - 2$ (5)

$ax^4 + bx^3 + cx^2 + 1 \equiv (x-1)^3\phi(x) + x - 2$

$\frac{d}{dx} \Rightarrow \quad 4ax^3 + 3bx^2 + 2cx \equiv (x-1)^3\phi'(x) + \phi(x)\cdot 3(x-1)^2\cdot 1 + 1$ (5)

$x=1 \Rightarrow \quad 4a+3b+2c = 1$ — (2) (5)

$\frac{d}{dx} \Rightarrow \quad 12ax^2 + 6bx + 2c = (x-1)^3\phi''(x) + \phi'(x)\cdot 3(x-1)^2\cdot 1 + 3(x-1)^2\phi'(x) + 3\phi(x)\cdot 2(x-1)\cdot 1$ (5)

$x=1 \Rightarrow \quad 12a+6b+2c = 0$ — (3) (5)

$6a+3b+c = 0$ — (3)

(3) − (2) $\Rightarrow \quad 8a+3b = -1$ — (4)

(3) − (1) $\Rightarrow \quad 5a+2b = 2$ — (5)

(4)2 − (5)3 $\Rightarrow \quad a = -2-6 \Rightarrow a = -8$

(5) $\Rightarrow \quad 2b = 2-5(-8) \Rightarrow b = 21$

(1) $\Rightarrow \quad c = -2+8-21 \Rightarrow c = -15$

3වම (10), 2ට (5)

$\boxed{40}$

(12) (a) A − 6, M − 1, R − 1, N − 2, Y − 1

අකුරු 6 සියල්ලම ගෙන සෑදිය හැකි සංකරණ $= \frac{11!}{6!\,2!}$ (10) $= 27720$ (5)

$\boxed{15}$

(i) AMARA යන කාණ්ඩයක් හා NAYANA යන කාණ්ඩයක් වශයෙන් ගෙන කාණ්ඩ දෙකේ පිළිවෙළ කිරීම $= 2! = 2$ (5)

AMARA වචනයේ අකුරු එම කාණ්ඩය තුළ පිළිවෙළ වීම $= \frac{5!}{3!} = 20$ (5)

NAYANA වචනයේ අකුරු එම කාණ්ඩය තුළ පිළිවෙළ වීම $= \frac{6!}{2!\,3!} = 60$ (5)

$\therefore$ අවශ්‍ය වචන සංඛ්‍යාව $= 2 \times 20 \times 60 = 2400$ (5)

$\boxed{20}$

(ii) වචන දෙකම A-අකුරු ඉවත් කළ විට ඉතුරු අකුරු 5 මැද එන පිළිවෙළ කිරීම $= \frac{5!}{2!}$ (5) $= 60$

-AAA (ඉතිරි අකුරු 5) AAA

වචන දෙකේ A අකුරු මුලට හා අගට පිළිවෙළ කිරීම $= 1$ (5)

$\therefore$ සෑදිය හැකි වචන සංඛ්‍යාව $= 60 \times 1 = 60$ (5)

$\boxed{15}$

(iii) A − 6
R − 1
N − 2

වෙනස් අකුරු කාණ්ඩ − 3

තේරීම් සංකරණ

(10) + (10)

	සංයෝජන	සංකරණ
අකුරු 4ම එකම වර්ගයෙන් තේරීම →	${}^1C_1 = 1$	$1\frac{4!}{4!} = 1$
අකුරු 3ක් එකම වර්ගයකින් හා අනෙක් වෙනස්ව තේරීම →	${}^1C_1 \times {}^2C_1 = 2$	$2\frac{4!}{3!} = 8$
අකුරු 2ක් සමානව, අනෙක් දෙක සමානව තේරීම →	${}^2C_2 = 1$	$1\frac{4!}{2!\,2!} = 6$
අකුරු 2ක් සමානව, අනෙක් දෙක වෙනස්ව තේරීම →	${}^2C_1 \times {}^2C_2 = 2$	$2\frac{4!}{2!} = 24$
අකුරු 4ම වෙනස්ව තේරීම →	0	

සංයෝජන $= 6$ (5) සංකරණ $= 39$ (5)

$\boxed{30}$

(b)

$$\frac{3r+1}{(r+1)(r+2)(r+3)} = \frac{A}{(r+2)} - \frac{B}{(r+1)(r+2)} - \frac{A}{(r+3)} + \frac{B}{(r+2)(r+3)}$$

$$= A\left(\frac{1}{r+2} - \frac{1}{r+3}\right) - B\left(\frac{1}{(r+1)(r+2)} - \frac{1}{(r+2)(r+3)}\right)$$

$$\frac{3r+1}{(r+1)(r+2)(r+3)} = \frac{A}{(r+2)(r+3)} - \frac{2B}{(r+1)(r+2)(r+3)} \quad 5$$

$$3r+1 = A(r+1) - 2B$$

$r:\ A = 3$ 5, $r^0:\ 1 = \frac{A}{3} - 2B \Rightarrow B = 1$ 5 $\boxed{15}$

$$\therefore U_r = \frac{3}{(r+2)} - \frac{1}{(r+1)(r+2)} - \frac{3}{(r+3)} + \frac{1}{(r+2)(r+3)} \quad 5$$

$$U_r = 3\left(\frac{1}{r+2} - \frac{1}{r+3}\right) - \left(\frac{1}{(r+1)(r+2)} - \frac{1}{(r+2)(r+3)}\right)$$

$$U_r = 3\left(V_r - V_{r+1}\right) - \left(W_r - W_{r+1}\right) \quad \therefore V_r = \frac{1}{r+2} \quad 5$$

V_r, W_r සඳහා

$W_r = \frac{1}{(r+1)(r+2)}$

$r=1 \Rightarrow U_1 = 3(V_1 - \cancel{V_2}) - (W_1 - \cancel{W_2})$

$r=2 \Rightarrow U_2 = 3(\cancel{V_2} - \cancel{V_3}) - (\cancel{W_2} - \cancel{W_3})$ 5

$\vdots$

$r=n-1 \Rightarrow U_{n-1} = 3(\cancel{V_{n-1}} - \cancel{V_n}) - (\cancel{W_{n-1}} - \cancel{W_n})$

$r=n \Rightarrow U_n = 3(\cancel{V_n} - V_{n+1}) - (\cancel{W_n} - W_{n+1})$ 5

$V_1 = \frac{1}{3}$

$V_{n+1} = \frac{1}{n+3}$

$W_1 = \frac{1}{2\cdot 3} = \frac{1}{6}$

$W_{n+1} = \frac{1}{(n+2)(n+3)}$

$$\sum_{r=1}^{n} U_r = 3\left(V_1 - V_{n+1}\right) - \left(W_1 - W_{n+1}\right) \quad 5$$

$$= 3\left(\frac{1}{3} - \frac{1}{n+3}\right) - \left(\frac{1}{6} - \frac{1}{(n+2)(n+3)}\right)$$

$$\sum_{r=1}^{n} U_r = 1 - \frac{1}{6} - \frac{3}{n+3} + \frac{1}{(n+2)(n+3)} \quad 5 \quad = \frac{5}{6} - \frac{3}{(n+3)} + \frac{1}{(n+2)(n+3)} \quad \boxed{30}$$

ශ්‍රේණිය අභිසාරී වේ. 5

$$\lim_{n\to\infty} \sum_{r=1}^{n} U_r = \frac{5}{6} - \lim_{n\to\infty}\left[\frac{3}{n+3} - \frac{1}{(n+2)(n+3)}\right] \quad 5$$

$$= \frac{5}{6} - \lim_{n\to\infty} \frac{3n+8}{(n+2)(n+3)} = \frac{5}{6} - \lim_{n\to\infty} \frac{3/n + 8/n^2}{(1+2/n)(1+3/n)}$$

$$\sum_{r=1}^{\infty} U_r = \frac{5}{6} - 0 = \frac{5}{6} \quad 5$$

එනම් ශ්‍රේණියේ $n \to \infty$ දී ඓක්‍යය සංඛ්‍යාවක පරිමිත වේ. එනම් ශ්‍රේණිය අභිසාරී වේ.

$\boxed{15}$

$$\sum_{r=n+1}^{2n} U_r = \sum_{r=1}^{2n} U_r - \sum_{r=1}^{n} U_r \quad (5)$$

$$= \frac{5}{6} - \frac{3}{(2n+3)} + \frac{1}{(2n+2)(2n+3)} - \left[\frac{5}{6} - \frac{3}{(n+3)} + \frac{1}{(n+2)(n+3)}\right] \quad (5)$$

$$\sum_{r=n+1}^{2n} U_r = \frac{-6n-6+1}{2(n+1)(2n+3)} + \frac{3n+6-1}{(n+2)(n+3)}$$

$$\sum_{r=n+1}^{2n} U_r = \frac{-(6n+5)}{2(n+1)(2n+3)} + \frac{(3n+5)}{(n+2)(n+3)} = \frac{(3n+5)}{(n+2)(n+3)} - \frac{(6n+5)}{2(n+1)(2n+3)}$$

$\boxed{10}$

(13) (a)

$$A + \lambda B = \begin{pmatrix} 4 & 5 & 6 \\ 2 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix} + \lambda \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 5 & 6 \\ 2 & 1 & 0 \end{pmatrix} \quad (5)$$

$$(5) \quad \begin{pmatrix} 1+2\lambda & 2+2\lambda & 3+2\lambda \\ -1+2\lambda & -2+2\lambda & -3+2\lambda \end{pmatrix} = \begin{pmatrix} 4 & 5 & 6 \\ 2 & 1 & 0 \end{pmatrix}$$

සමීකරණ 6ට (10) හෝ 0
$$\left\{ \begin{array}{lll} 1+2\lambda = 4, & 2+2\lambda = 5, & 3+2\lambda = 6 \\ -1+2\lambda = 2, & -2+2\lambda = 1, & -3+2\lambda = 0 \end{array} \right\} \lambda = \frac{3}{2} \quad (5)$$

$\boxed{25}$

$$C = (A+\lambda B)B^T = \begin{pmatrix} 4 & 5 & 6 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ 2 & 2 \\ 2 & 2 \end{pmatrix} \quad (5)$$

$$C = \begin{pmatrix} 30 & 30 \\ 6 & 6 \end{pmatrix}_{2\times 2} \quad (5)$$

$\boxed{10}$

$$D^T = (1 \; 2) \Rightarrow D = \begin{pmatrix} 1 \\ 2 \end{pmatrix}_{2\times 1} \quad (5)$$

$$M = CD = \begin{pmatrix} 30 & 30 \\ 6 & 6 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (5)$$

$$M = \begin{pmatrix} 90 \\ 18 \end{pmatrix}_{2\times 1} \quad (5)$$

$\boxed{15}$

(b) $z_1 = x_1 + iy_1 \;,\; x_1, y_1 \in \mathbb{R} \;,\; z_2 = x_2 + iy_2 \;,\; x_2, y_2 \in \mathbb{R}$ ලෙස ගනිමු.

(i) $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$

$\overline{z_1 + z_2} = (x_1 + x_2) - i(y_1 + y_2)$ (5)

$= (x_1 - iy_1) + (x_2 - iy_2)$

$\overline{z_1 + z_2} = \overline{z}_1 + \overline{z}_2$ (5) $\boxed{10}$

(ii) $z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2) = x_1 x_2 + \underbrace{i^2}_{-1} y_1 y_2 + i(x_1 y_2 + y_1 x_2)$ (5)

$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + y_1 x_2)$

$\overline{z_1 z_2} = (x_1 x_2 - y_1 y_2) - i(x_1 y_2 + y_1 x_2)$

$= (x_1 - iy_1)(x_2 - iy_2)$

$\overline{z_1 z_2} = \overline{z}_1 \overline{z}_2$ (5) $\boxed{10}$

(iii) $|z_1| = \sqrt{x_1^2 + y_1^2} = \sqrt{(x_1 + iy_1)(x_1 - iy_1)}$

$|z_1|^2 = (x_1 + iy_1)(x_1 - iy_1)$ (5)

$|z_1|^2 = z_1 \cdot \overline{z}_1$ (5) $\boxed{10}$

(iv) $z_1 \overline{z}_2 + z_2 \overline{z}_1 = (x_1 + iy_1)(x_2 - iy_2) + (x_2 + iy_2)(x_1 - iy_1)$

$= x_1 x_2 - \underbrace{i^2}_{-1} y_1 y_2 + i(y_1 x_2 - x_1 y_2) + x_1 x_2 - \underbrace{i^2}_{-1} y_1 y_2 + i(y_2 x_1 - x_2 y_1)$ (5)

$z_1 \overline{z}_2 + z_2 \overline{z}_1 = 2(x_1 x_2 + y_1 y_2) = 2\,\mathrm{Re}(z_1 \overline{z}_2)$ (5) $\boxed{10}$

$z_1 \neq -z_2$ වන විට $\left|\dfrac{1 + z_1 z_2}{z_1 + z_2}\right| = 1 \;,\; |z_1| = |z_2| = 1$

$\left|\dfrac{1 + z_1 z_2}{z_1 + z_2}\right|^2 = 1$ (5) $\Rightarrow \left(\dfrac{1 + z_1 z_2}{z_1 + z_2}\right)\left(\overline{\dfrac{1 + z_1 z_2}{z_1 + z_2}}\right) = 1$ (5)

$\left(\dfrac{1 + z_1 z_2}{z_1 + z_2}\right)\left(\dfrac{1 + \overline{z}_1 \overline{z}_2}{\overline{z}_1 + \overline{z}_2}\right) = 1$ (5)

$\therefore 1 + \overline{z}_1 \overline{z}_2 + z_1 z_2 + z_1 \overline{z}_1 z_2 \overline{z}_2 = z_1 \overline{z}_1 + z_1 \overline{z}_2 + z_2 \overline{z}_1 + z_2 \overline{z}_2$

$1 + \overline{z}_1 \overline{z}_2 + z_1 z_2 + \underbrace{|z_1|^2}_{1} \underbrace{|z_2|^2}_{1} = \underbrace{|z_1|^2}_{1} + \underbrace{(z_1 \overline{z}_2 + z_2 \overline{z}_1)}_{2\mathrm{Re}(z_1 \overline{z}_2)} + \underbrace{|z_2|^2}_{1}$ (5)

$2 + z_1 z_2 + \overline{z}_1 \overline{z}_2 = 2 + 2\,\mathrm{Re}(z_1 \overline{z}_2)$ (5)

$\therefore z_1 z_2 + \overline{z}_1 \overline{z}_2 = 2\,\mathrm{Re}(z_1 \overline{z}_2)$ $\boxed{25}$

ද්විපද ප්‍රසාරණයෙන්,

(c) $(\cos\theta + i\sin\theta)^4 = \cos^4\theta + 4\cos^3\theta \cdot i\sin\theta + 6\cos^2\theta\, \underbrace{i^2}_{-1}\sin^2\theta + 4\cos\theta\, \underbrace{i^3}_{-i}\sin^3\theta + \underbrace{i^4}_{1}\sin^4\theta$

$(\cos\theta + i\sin\theta)^4 = \cos^4\theta + \sin^4\theta - 6\sin^2\theta\cos^2\theta + i(4\sin\theta\cos^3\theta - 4\cos\theta\sin^3\theta)$ — (1)

(5)

ද' මුවාවර් ප්‍රමේයයෙන්,

$(\cos\theta + i\sin\theta)^4 = \cos 4\theta + i\sin 4\theta$ — (2)

(5)

(1) = (2) => තාත්වික කොටස් සමාන කිරීමෙන් =>

(b) — $\cos 4\theta = \cos^4\theta + \sin^4\theta - 6\sin^2\theta\cos^2\theta$ (5)

අතාත්වික කොටස් සමාන කිරීමෙන්,

(a) — $\sin 4\theta = 4\sin\theta\cos^3\theta - 4\cos\theta\sin^3\theta$ (5)

(a)/(b) => $\tan 4\theta = \dfrac{4\sin\theta\cos^3\theta - 4\cos\theta\sin^3\theta}{\sin^4\theta + \cos^4\theta - 6\sin^2\theta\cos^2\theta}$ (5)

$= \dfrac{\dfrac{4\sin\theta\cos^3\theta}{\cos^4\theta} - \dfrac{4\cos\theta\sin^3\theta}{\cos^4\theta}}{\dfrac{\sin^4\theta + \cos^4\theta - 6\sin^2\theta\cos^2\theta}{\cos^4\theta}}$ (5)

$\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{\tan^4\theta + 1 - 6\tan^2\theta}$ (5) $= \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$ $\boxed{35}$

(14) (a) $f(x) = \dfrac{x^2 - 1}{(x+2)^2}, \quad x \neq -2$

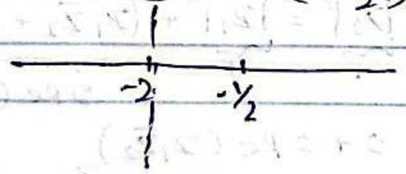
$f'(x) = \dfrac{(x+2)^2 2x - (x^2-1)\cdot 2(x+2)\cdot 1}{\left[(x+2)^2\right]^2}$ (10) or 0

$f'(x) = \dfrac{2x(x+2) - 2(x^2-1)}{(x+2)^3} = \dfrac{2(x^2 + 2x - x^2 + 1)}{(x+2)^3}$ (5)

$f'(x) = \dfrac{2(2x+1)}{(x+2)^3}, \quad x \neq -2$ $\boxed{15}$

$f'(x) = 0 \Rightarrow x = -\frac{1}{2}$ (5), $x = -\frac{1}{2}$ විට $y = f(x) = \dfrac{\frac{1}{4} - 1}{(2 - \frac{1}{2})^2} = -\dfrac{1}{3}$

හැරුම් ලක්ෂ්‍යය $\left(-\frac{1}{2}, -\frac{1}{3}\right)$ (5)



$\lim_{x \to \pm\infty} f(x) = 1$ ∴ $y = 1$ තිරස් ස්පර්ශෝන්මුඛයකි.

(5)

$\lim_{x \to -2^-} f(x) = \infty$

$\lim_{x \to -2^+} f(x) = \infty$ ∴ $x = -2$ සිරස් ස්පර්ශෝන්මුඛයකි.

(5)

$$f''(x) = \frac{2(1-4x)}{(x+2)^4}, \quad x \neq -2$$

$f''(x) = 0 \Rightarrow 1-4x = 0 \Rightarrow x = 1/4$ (5)

$x = \frac{1}{4}$ දී $y = -\frac{15}{81}$ $\therefore \left(\frac{1}{4}, -\frac{15}{81}\right)$ නතිවර්තන ලක්ෂ්‍යයකි. (5)

$-2 \quad 1/4$

	$-\infty < x < -2$	$-2 < x < -1/2$	$-\frac{1}{2} < x < \infty$
$f'(x)$ ලකුණ	(+)	(−)	(+)
$f(x)$	ඒකාකීව වැඩිවේ	ඒකාකීව අඩුවේ	ඒකාකීව වැඩිවේ

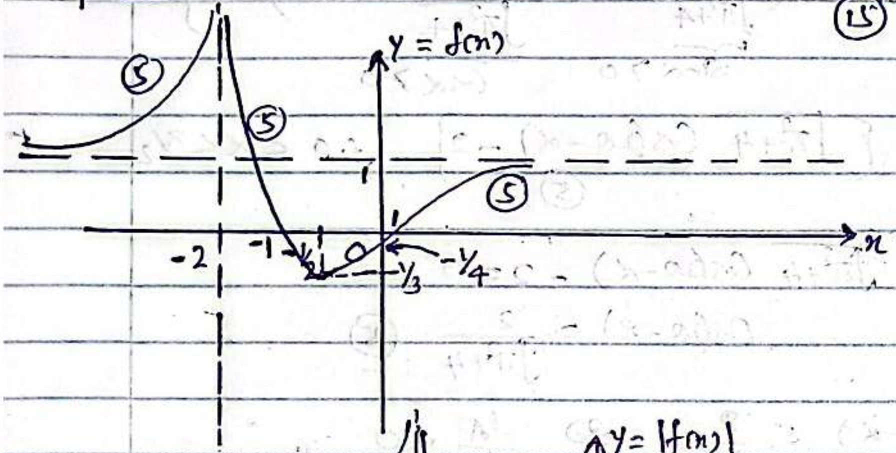
(15)

$\therefore$ $x = -1/2$ දී $y = -1/3$ අවම වේ. අවම ලක්ෂ්‍යය $\left(-\frac{1}{2}, -\frac{1}{3}\right)$

	$-\infty < x < -2$	$-2 < x < \frac{1}{4}$	$\frac{1}{4} < x < \infty$
$f''(x)$ ලකුණ	(+)	(+)	(−)
අවතලතාවය	උඩු අතට	උඩු අතට	යටි අතට

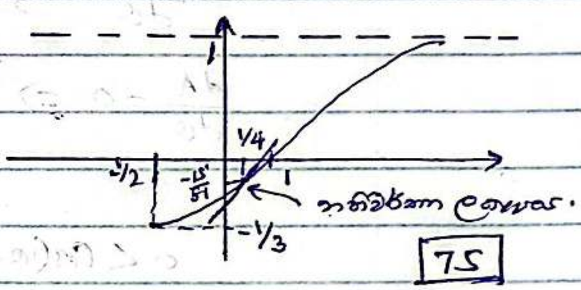
(15)

$x = \frac{1}{4}$ දී අවතලතාවය වෙනස් කරන ලක්ෂ්‍යයකි.
$\therefore$ නතිවර්තන ලක්ෂ්‍යය $\left(\frac{1}{4}, -\frac{15}{81}\right)$ වේ.



$y = f(x) = 0$

$x^2 - 1 = 0 \Rightarrow x = \pm 1$



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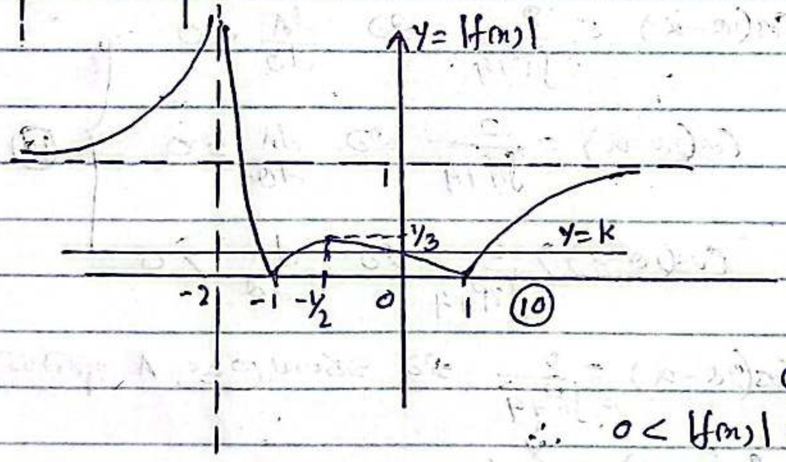


$|f(x)| = k$

$y = k$

$0 < |f(x)| < \frac{1}{3}$ පරාසය තුළ

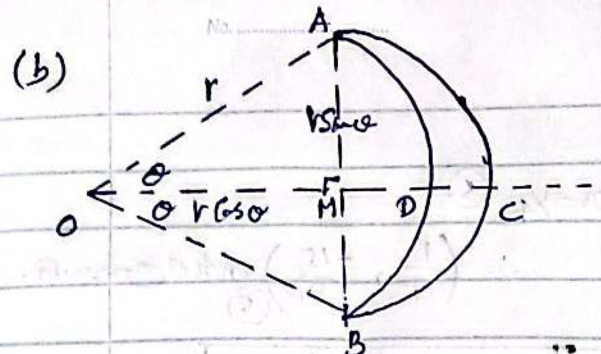
$y = k$ සරල $y = f(x)$ ඡේදන ලක්ෂ්‍ය 4කි. (5)

$\therefore$ $0 < |f(x)| < \frac{1}{3}$ තුළ $|f(x)| = k, \; k \in \mathbb{R}^+$

සමීකරණයේ විසඳුම් ගණන 4කි.

15

(b)



$AOB \ \Delta$ වර්ග $= \frac{1}{2} 2r\sin\theta \cdot r\cos\theta$

$= r^2 \sin\theta\cos\theta$

OAB කේන්ද්‍රික ඛ. වර්ග $= \frac{1}{2} r^2 \cdot 2\theta = r^2\theta$

ACB අර්ධ වෘත්තයේ වර්ගඵලය $= \frac{\pi r^2 \sin^2\theta}{2}$

ඉතිරි වර්ගඵලය $= ACB$ වර්ග $-$ (OAB කේ. ඛ. වර්ග $- AOB \ \Delta$ වර්ග)

$$A = \frac{\pi r^2}{2}\sin^2\theta - r^2\theta + r^2\sin\theta\cos\theta \quad (5)$$

$$A = \frac{r^2}{2}\left(\pi\sin^2\theta - 2\theta + \underbrace{2\sin\theta\cos\theta}_{\sin 2\theta}\right)$$

$$\frac{dA}{d\theta} = \frac{r^2}{2}\left(\pi \cdot \underbrace{2\sin\theta\cos\theta}_{\sin 2\theta} - 2 + 2\cos 2\theta\right)$$

$$\frac{dA}{d\theta} = \frac{r^2}{2}\left(\pi\sin 2\theta + 2\cos 2\theta - 2\right) \quad (5)$$

$$\frac{dA}{d\theta} = \frac{r^2}{2}\left[\sqrt{\pi^2+4}\left(\underbrace{\frac{\pi}{\sqrt{\pi^2+4}}}_{\sin\alpha > 0}\sin 2\theta + \underbrace{\frac{2}{\sqrt{\pi^2+4}}}_{\cos\alpha > 0}\cos 2\theta\right) - 2\right]$$

$$\frac{dA}{d\theta} = \frac{r^2}{2}\left[\sqrt{\pi^2+4}\cos(2\theta-\alpha) - 2\right] \quad (5), \ \therefore 0 < \alpha < \pi/2$$

$$\frac{dA}{d\theta} = 0 \Rightarrow \sqrt{\pi^2+4}\cos(2\theta-\alpha) - 2 = 0$$

$$\cos(2\theta-\alpha) = \frac{2}{\sqrt{\pi^2+4}} \quad (5)$$

$0 < \cos(2\theta-\alpha) < \frac{2}{\sqrt{\pi^2+4}}$ විට $\frac{dA}{d\theta} < 0$

$\cos(2\theta-\alpha) = \frac{2}{\sqrt{\pi^2+4}}$ විට $\frac{dA}{d\theta} = 0$ (10)

$\cos(2\theta-\alpha) > \frac{2}{\sqrt{\pi^2+4}}$ විට $\frac{dA}{d\theta} > 0$

නිසා, $\cos(2\theta-\alpha) = \frac{2}{\sqrt{\pi^2+4}}$ විට වර්ගඵලය A අවම වේ.

$$\cos(2\theta-\alpha) = \cos\alpha$$

$$2\theta - \alpha = \alpha \quad (5)$$

$$\theta = \alpha \qquad \therefore \theta = \cos^{-1}\frac{2}{\sqrt{\pi^2+4}} \text{ විට } A \text{ අවම වේ.}$$

$\boxed{35}$

(15) (a) $\displaystyle\int \frac{dx}{(ax^2+b)(bx+a)}$

$a=0$ හා $b\neq 0$ විට $\displaystyle\int \frac{dx}{(ax^2+b)(bx+a)} = \int \frac{dx}{b^2x}$ (5) $= \frac{1}{b^2}\ln|x| + C_1$ (5) $\boxed{10}$

$b=0$ හා $a\neq 0$ විට $\displaystyle\int \frac{dx}{(ax^2+b)(bx+a)} = \int \frac{dx}{a^2x^2}$ (5) $= \frac{-1}{a^2x} + C_2$ (5) $\boxed{10}$

$a\neq 0$ හා $b\neq 0$ විට:

$$\frac{1}{(ax^2+b)(bx+a)} = \frac{Ax+B}{ax^2+b} + \frac{C}{bx+a} \quad (5)$$

$$1 = (Ax+B)(bx+a) + C(ax^2+b)$$

$x^2:\ 0 = Ab + aC \Rightarrow A = -\frac{a}{b}C$

$x:\ 0 = aA + bB \Rightarrow A = -\frac{b}{a}B$

$x^0:\ 1 = Ba + bC \Rightarrow 1 = Ba + \frac{b}{a}(-bA)$

$1 = Ba - \frac{b^2}{a}\left(-\frac{b}{a}\right)B \Rightarrow 1 = \frac{B}{a^2}(a^3+b^3)$

(5) $B = \dfrac{a^2}{a^3+b^3}$, $A = -\dfrac{b}{a}\cdot\dfrac{a^2}{(a^3+b^3)} = \dfrac{-ab}{a^3+b^3}$ (5)

$C = -\dfrac{b}{a}\cdot\dfrac{(-ab)}{(a^3+b^3)} = \dfrac{b^2}{a^3+b^3}$ (5)

$$\frac{1}{(ax^2+b)(bx+a)} = \frac{a(-bx+a)}{(a^3+b^3)(ax^2+b)} + \frac{b^2}{(a^3+b^3)(bx+a)} \quad (5)$$

$$\int \frac{dx}{(ax^2+b)(bx+a)} = \frac{a}{(a^3+b^3)}\int \frac{-bx+a}{ax^2+b}\,dx + \frac{b^2}{(a^3+b^3)}\int \frac{dx}{(bx+a)} \quad (5)$$

$$= \frac{a}{(a^3+b^3)}\left[\frac{-b}{2a}\int \frac{2ax}{ax^2+b}\,dx + \int \frac{dx}{x^2+\sqrt{\frac{b}{a}}^{\,2}}\right] + \frac{b}{(a^3+b^3)}\int \frac{b}{bx+a}\,dx \quad (5)\ (5)\ (5)$$

$$\int \frac{dx}{(ax^2+b)(bx+a)} = \frac{a}{(a^3+b^3)}\left[\frac{-b}{2a}\ln|ax^2+b| + \sqrt{\frac{a}{b}}\tan^{-1}\left(x\sqrt{\frac{a}{b}}\right)\right] + \frac{b}{(a^3+b^3)}\ln|bx+a| + C' \quad (5)\ (5)\ (5)$$

ස්වේච්ඡ නියතයකි (5)

$\boxed{65}$

(b) $\displaystyle\int_0^{\sqrt{\pi/6}} x^5 \cos x^2\,dx = \frac{1}{2}\int_0^{\pi/6} t^2 \cos t\,dt$ ⑤

⑤ $x^2 = t \Rightarrow x=0$ විට $t=0$

$2x\dfrac{dx}{dt} = 1$ $\quad x=\sqrt{\frac{\pi}{6}}$ විට $t=\frac{\pi}{6}$

$x\dfrac{dx}{dt} = \dfrac{1}{2}$ ⑤

$$= \frac{1}{2}\int_0^{\pi/6} t^2 \frac{d}{dt}(\sin t)\,dt$$

$$= \frac{1}{2}\left[t^2 \sin t \Big|_0^{\pi/6} - \int_0^{\pi/6} \sin t \cdot 2t\,dt \right] ⑤$$

$$= \frac{1}{2}\left[\frac{\pi^2}{36}\sin\frac{\pi}{6} - 0 + 2\int_0^{\pi/6} t\,\frac{d}{dt}(+\cos t)\,dt \right] ⑤$$

$$= \frac{\pi^2}{144} + t\cos t\Big|_0^{\pi/6} - \int_0^{\pi/6} \cos t\,\underbrace{\frac{d}{dt}(t)}_{1}\,dt ⑤$$

$$= \frac{\pi^2}{144} + \frac{\pi}{6}\cos\frac{\pi}{6} - 0 - \sin t\Big|_0^{\pi/6}$$

$$= \frac{\pi^2}{144} + \frac{\sqrt{3}\pi}{12} - (\sin\pi/6 - 0)$$

$$\int_0^{\sqrt{\pi/6}} x^5 \cos x^2\,dx = \frac{\pi^2}{144} + \frac{\sqrt{3}\pi}{12} - \frac{1}{2} \quad ⑤$$

$\boxed{35}$

(c) $I = \displaystyle\int_0^a f(x)\,dx = -\int_a^0 f(a-y)\,dy$ ⑤

$x = a-y \Rightarrow x=0$ විට $y=a$

$\dfrac{dx}{dy} = -1$ ⑤ $\quad x=a$ විට $y=0$

$I = \displaystyle\int_0^a f(a-y)\,dy = \int_0^a f(a-x)\,dx = J$

$\boxed{10}$

$$I_m = \int_0^{\pi/2} \frac{dx}{1+\cot^m(x)} \quad —①$$

$$I_m = \int_0^{\pi/2} \frac{dx}{1+\cot^m\left(\frac{\pi}{2}-x\right)} ⑤ = \int_0^{\pi/2} \frac{dx}{1+\tan^m x} = \int_0^{\pi/2} \frac{\cot^m x}{1+\cot^m x}\,dx \quad —②$$

① + ② $\Rightarrow$ $2I_m = \displaystyle\int_0^{\pi/2} dx = x\Big|_0^{\pi/2} = \pi/2 \Rightarrow I_m = \frac{\pi}{4}$ ⑤

$\boxed{10}$

$$\int_0^{\pi} \frac{dx}{1+\cot^m\left(\frac{x}{2}\right)} = 2\int_0^{\pi/2} \frac{dy}{1+\cot^m y} \quad ⑤$$

⑤ $\dfrac{x}{2} = y$ ආදේශයෙන් $\quad x=0$ විට $y=0$

$x = 2y$ $\quad x=\pi$ විට $y=\pi/2$

$\dfrac{dx}{dy} = 2$

$= 2I_m$

$= 2\cdot\dfrac{\pi}{4}$

$$\int_0^{\pi} \frac{dx}{1+\cot^m\left(\frac{x}{2}\right)} = \frac{\pi}{2}$$

[illegible]

$\boxed{10}$

(16.) $y = m_1x + c_1$ — (1) $\quad y = m_2x + c_2$ — (2) $\quad y = m_3x + c_3$ — (3)

$(1) = (2) \Rightarrow m_1x + c_1 = m_2x + c_2$

$x = \dfrac{c_2 - c_1}{m_1 - m_2}$, $m_1 \neq m_2$ (5) එවිට $y = \dfrac{m_1(c_2 - c_1)}{m_1 - m_2} + c_1$

$y = \dfrac{m_1c_2 - m_2c_1}{m_1 - m_2}$ (5)

(1) හා (2) හි ඡේදන ලක්ෂ්‍යය $\left(\dfrac{c_2 - c_1}{m_1 - m_2}, \dfrac{m_1c_2 - m_2c_1}{m_1 - m_2}\right)$ වේ. රේඛා 3 සංගාමී

විට, මෙම ලක්ෂ්‍යය (3) මත පිහිටයි.

$\therefore \dfrac{m_1c_2 - m_2c_1}{(m_1 - m_2)} = \dfrac{m_3(c_2 - c_1)}{(m_1 - m_2)} + c_3$ (5)

$m_1c_2 - m_2c_1 = m_3(c_2 - c_1) + c_3(m_1 - m_2)$

$m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$ — (A)

(5) $\boxed{20}$

AB රේඛාව $y = m_3x + c_3$ යැයි ගනිමු.

$y = 2x - 1$, $y = 3x + 1$ හා $y = m_3x + c_3$, A හි ඡේදනය වේ,

(5) $m_1 = 2$, $m_2 = 3$, $m_3 = m_3$, $c_1 = -1$, $c_2 = 1$, $c_3 = c_3$ ලෙස (A) $\Rightarrow$

$2(1 - c_3) + 3(c_3 + 1) + m_3(-1 - 1) = 0$ (5)

$c_3 - 2m_3 = -5$ — (1)

$y = -3x + 4$, $y = 4x - 2$ හා $y = m_3x + c_3$, B හිදී ඡේදනය වේ

(5) $m_1 = -3$, $m_2 = 4$, $m_3 = m_3$, $c_1 = 4$, $c_2 = -2$, $c_3 = c_3$ ලෙස (A) $\Rightarrow$

$-3(-2 - c_3) + 4(c_3 - 4) + m_3(4 + 2) = 0$ (5)

$7c_3 + 6m_3 = 10$ — (2)

$(1) \cdot 3 + (2) \Rightarrow 10c_3 = -5 \Rightarrow c_3 = -\frac{1}{2}$ (5) $\quad (1) \Rightarrow 2m_3 = -\frac{1}{2} + 5 = \frac{9}{2}$

$m_3 = \frac{9}{4}$ (5)

$\therefore$ AB රේඛාවේ සමීකරණය $y = \frac{9}{4}x - \frac{1}{2} \Rightarrow 9x - 4y - 2 = 0$ (5) $\boxed{35}$

$y = 2x - 1$ — (1) $\quad y = 3x + 1$ — (2) $\quad y = -3x + 4$ — (3) $\quad y = 4x - 2$ — (4)

$(1) = (2) \Rightarrow 2x - 1 = 3x + 1$

$x = -2$ විට $y = -5$

$\therefore A(-2, -5)$ (5)

$(3) = (4) \Rightarrow -3x + 4 = 4x - 2$

$7x = 6 \Rightarrow x = 6/7$

$x = 6/7$ විට $y = -\frac{18}{7} + 4 = \frac{10}{7}$

$\therefore B\left(\frac{6}{7}, \frac{10}{7}\right)$ (5)

AB විෂ්කම්භය වන වෘත්තයේ සමීකරණය $(x+2)\left(x - \frac{6}{7}\right) + (y+5)\left(y - \frac{10}{7}\right) = 0$ (5)

$x^2 + \frac{8}{7}x - \frac{12}{7} + y^2 + \frac{25}{7}y - \frac{50}{7} = 0$ (5)

$7x^2 + 7y^2 + 8x + 25y - 62 = 0$ $\boxed{20}$

AB විෂ්කම්භය වන වෘත්තය $S \equiv 7x^2+7y^2+8x+25y-62=0$

$l = 2x-y-1=0$

$S=0$ හා $l=0$ හි ඡේදන ලක්ෂ්‍ය හරහා යන ඕනෑම වෘත්තයක්

$S+\lambda l=0$ ආකාර වේ. (10)

$$7x^2+7y^2+8x+25y-62+\lambda(2x-y-1)=0$$

$$(5) \quad 7x^2+7y^2+(8+2\lambda)x+(25-\lambda)y-62-\lambda=0 \quad \text{—} (*)$$

$$(5) \quad x^2+y^2+\frac{2}{7}(4+\lambda)x+\frac{1}{7}(25-\lambda)y-\frac{1}{7}(62+\lambda)=0 \quad \text{—} (B)$$

$$(5) \quad g_1=\frac{1}{7}(4+\lambda), \quad f_1=\frac{1}{14}(25-\lambda) \ (5), \quad c_1=-\frac{1}{7}(62+\lambda) \ (5)$$

$$x^2+y^2+2x-2y+1=0 \quad \text{—} (C)$$

$$g_2=1, \quad f_2=-1, \quad c_2=1 \quad (10)$$

(B) හා (C) ප්‍රලම්බ නම්,

$2g_1g_2+2f_1f_2=c_1+c_2$ විය යුතුය. (10)

$$\frac{2}{7}(4+\lambda)\cdot 1+\frac{2}{14}(25-\lambda)\cdot(-1)=-\frac{1}{7}(62+\lambda)+1 \quad (10)$$

$$8+2\lambda-25+\lambda=-62-\lambda+7$$

$$4\lambda=-38 \Rightarrow \lambda=-\frac{19}{2} \quad (5)$$

$\therefore$ අවශ්‍ය වෘත්තයේ සමීකරණය $\lambda=-\frac{19}{2}$ ආදේශ (*) $\Rightarrow$

$$(5) \quad 7x^2+7y^2+(8-19)x+\left(25+\frac{19}{2}\right)y-62+\frac{19}{2}=0$$

$$14x^2+14y^2-22x+69y-105=0$$

$\boxed{75}$

(17) (a)

$$\sin x+\cos x = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)$$

$$= \sqrt{2}\left(\sin\frac{\pi}{4}\sin x+\cos\frac{\pi}{4}\cos x\right) \quad (5)$$

$$\sin x+\cos x = \sqrt{2}\cos\left(x-\frac{\pi}{4}\right) \ (5) = R\cos(x-\theta), \quad R=\sqrt{2}, \ \theta=\frac{\pi}{4}$$

$(R>0, \ 0<\theta<\pi/2)$ (5) ඔක්කොම

$\boxed{15}$

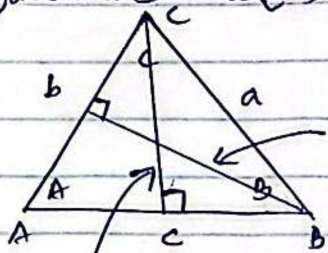
$$x \to -x \Rightarrow (5) \quad \sin(-x)+\cos(-x)=\sqrt{2}\cos\left(-x-\frac{\pi}{4}\right) \quad (5)$$

$$-\sin x+\cos x = \sqrt{2}\cos\left(x+\frac{\pi}{4}\right)$$

$$\cos x-\sin x = \sqrt{2}\cos\left(x+\frac{\pi}{4}\right) = R\cos(x+\theta)$$

$\boxed{10}$

ABC සුළු කෝණී Δ වන විට



$a\sin C = c\sin A$

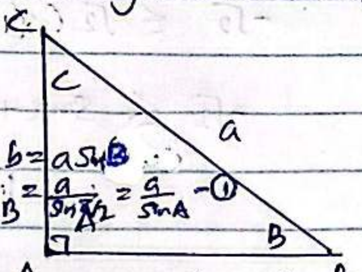
$\frac{a}{\sin A} = \frac{c}{\sin C}$ — ②

$b\sin A = a\sin B$

(5)

$\frac{a}{\sin A} = \frac{b}{\sin B}$ — ①

ABC සෘජුකෝණී Δ වන විට



$b = a\sin B$

$\frac{b}{\sin B} = \frac{a}{\sin \pi/2} = \frac{a}{\sin A}$ — ①

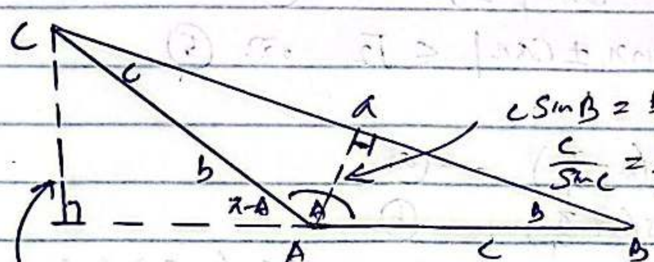
$c = a\sin C$

(5)

$\frac{a}{1} = \frac{c}{\sin C}$

$\frac{a}{\sin A} = \frac{c}{\sin C}$ — ②

ABC මහා කෝණී Δ වන විට



$c\sin B = b\sin C$

$\frac{c}{\sin C} = \frac{b}{\sin B}$ — ②

$b\underbrace{\sin(\pi - A)}_{\sin A} = a\sin B$

(5)

$\frac{a}{\sin A} = \frac{b}{\sin B}$ — ①

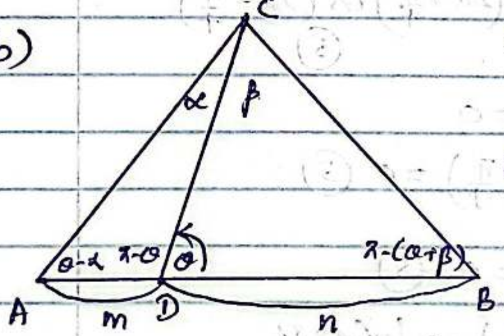
සයින් සූත්‍රය

① හා ② න්

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ වේ

$\boxed{15}$

(b)



$ADC \ \Delta \Rightarrow \frac{DC}{\sin(\theta - \alpha)} = \frac{AD}{\sin\alpha}$ — ① (5)

$BDC \ \Delta \Rightarrow \frac{DC}{\underbrace{\sin[\pi - (\theta+\beta)]}_{\sin(\theta+\beta)}} = \frac{DB}{\sin\beta}$ — ② (5)

$\frac{①}{②} \Rightarrow \frac{\sin(\theta+\beta)}{\sin(\theta-\alpha)} = \left(\frac{AD}{DB}\right)\frac{\sin\beta}{\sin\alpha}$

$AD = mk$
$DB = nk$
$\frac{AD}{DB} = \frac{m}{n}$

$\frac{\sin\theta\cos\beta + \cos\theta\sin\beta}{\sin\theta\cos\alpha - \cos\theta\sin\alpha} = \frac{m}{n}\frac{\sin\beta}{\sin\alpha}$ (5)

$n\sin\alpha\sin\theta\cos\beta + n\sin\alpha\cos\theta\sin\beta = m\sin\beta\sin\theta\cos\alpha - m\sin\beta\cos\theta\sin\alpha$

$\frac{(m+n)\sin\alpha\cos\theta\sin\beta}{\sin\alpha\sin\theta\sin\beta} = \frac{m\sin\beta\sin\theta\cos\alpha}{\sin\alpha\sin\theta\sin\beta} - \frac{n\sin\alpha\sin\theta\cos\beta}{\sin\alpha\sin\theta\sin\beta}$ (5)

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

$\boxed{20}$

(i) $-1 \leq \cos(x-\frac{\pi}{4}) \leq 1$

$-\sqrt{2} \leq \sqrt{2}\cos(x-\frac{\pi}{4}) \leq \sqrt{2}$ (5)

$-\sqrt{2} \leq \sin x + \cos x \leq \sqrt{2}$ (5)

$\therefore \; |\sin x + \cos x| \leq \sqrt{2}$ — (1)

$-1 \leq \cos(x+\frac{\pi}{4}) \leq 1$

$-\sqrt{2} \leq \sqrt{2}\cos(x-\frac{\pi}{4}) \leq \sqrt{2}$

$-\sqrt{2} \leq \sin x - \cos x \leq \sqrt{2}$ (5)

$\therefore \; |\sin x - \cos x| \leq \sqrt{2}$ — (2)

(1) හා (2) අනුව $|\sin x \pm \cos x| \leq \sqrt{2}$ වේ. (5) $\boxed{20}$

(ii) $\sin x + \cos x = \sqrt{2}\cos(x-\frac{\pi}{4})$ — (a)

$-\sin x + \cos x = \sqrt{2}\cos(x+\frac{\pi}{4})$ — (b)

(a) × (b) $\Rightarrow$ $(\sin x + \cos x)(-\sin x + \cos x) = 2\cos(x+\frac{\pi}{4})\cos(x-\frac{\pi}{4})$ (5)

$\cos^2 x - \sin^2 x = 2\cos(x+\frac{\pi}{4})\cos(x-\frac{\pi}{4})$ (5) $\boxed{10}$

$x = 2x \Rightarrow$ (5) $\cos^2 2x - \sin^2 2x = 2\cos(2x+\frac{\pi}{4})\cos(2x-\frac{\pi}{4})$ (5)

$\cos^2 2x - \sin^2 2x + \cos(2x+\frac{\pi}{4}) = 0$

$2\cos(2x+\frac{\pi}{4})\cos(2x-\frac{\pi}{4}) + \cos(2x+\frac{\pi}{4}) = 0$ (5)

$\cos(2x+\frac{\pi}{4})\left[2\cos(2x-\frac{\pi}{4})+1\right] = 0$ (5)

$\cos(2x+\frac{\pi}{4}) = 0 = \cos\pi/2$ හෝ $\cos(2x-\frac{\pi}{4}) = -\frac{1}{2} = \cos 2\pi/3$

$2x+\frac{\pi}{4} = 2n\pi \pm \frac{\pi}{2}$ $\qquad$ $2x-\frac{\pi}{4} = 2m\pi \pm \frac{2\pi}{3}$

(5) $x = n\pi \pm \frac{\pi}{4} - \frac{\pi}{8}$, $n \in \mathbb{Z}$. $\qquad$ (5) $x = m\pi \pm \frac{\pi}{3} + \frac{\pi}{8}$, $m \in \mathbb{Z}$. $\boxed{30}$

(b) සාමාන්‍ය ABC ත්‍රිකෝණයක් සඳහා සම්මත අංකනය අනුව

සයින් සූත්‍රය $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (5) $\boxed{05}$

No. .... Date. ....

(c) $\underbrace{\tan^{-1}(\cos 2x)}_{\alpha} + \underbrace{\tan^{-1}(\sin x)}_{\beta} = \frac{\pi}{4}$

$\tan\alpha = \cos 2x \quad , \quad \tan\beta = \sin x \quad , \quad -\frac{\pi}{2} < \alpha, \beta < \frac{\pi}{2}$

$\alpha + \beta = \frac{\pi}{4}$ 5

$\tan(\alpha+\beta) = \tan\frac{\pi}{4}$

$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 1 \Rightarrow \tan\alpha + \tan\beta = 1 - \tan\alpha\tan\beta$ 5

$\cos 2x + \sin x = 1 - \cos 2x \sin x$

$1 - 2\sin^2 x + \sin x = 1 - \sin x(1 - 2\sin^2 x)$

$4\sin^2 x - 2\sin x = 0$

$2\sin x(2\sin x - 1) = 0$ 5

$\sin x = 0$ and $\sin x = \frac{1}{2} = \sin\pi/6$

$x = n\pi \qquad x = m\pi + (-1)^m\frac{\pi}{6}$

5 $\quad n \in \mathbb{Z} \quad , m \in \mathbb{Z}$ 5 $\quad \boxed{25}$